



**2024 ENGINEERING INSTITUTION OF ZAMBIA  
SYMPOSIUM**

**Development of 2D design charts for rock slopes  
susceptible to slide-head toppling failure.**

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## Introduction

- Slide-head toppling failure is a secondary category of toppling which involves both sliding and toppling of rock blocks on a slope
- Analysis of this failure requires solving laborious complex equations
  - In determining toppling and sliding of rock blocks
- However, design charts prove to be handy in analyzing such failures
- A number of design charts have been developed for toppling failure and other failure mechanisms (wedge, circular, planar)
- Most design charts are developed based on the limit equilibrium equations
- Furthermore, previous design charts for toppling have been developed based on;



## Introduction Cont'

- discontinuity friction angles
- the height to width ratio
- slope angles, as well as inter-block forces (Zanbak, 1983; Cruden, 1989; Yagoda-Biran and Hatzor, 2013)
- Further research observations have noted that in some physical circumstances the failure plane for toppling blocks may not be regular as previously assumed (Zuo et al., 2005; Cai, 2013; Bowa and Xia, 2018; Bowa and Gong, 2021)
- Due to among other geological conditions within the rockmass, the failure plane may counter-tilt and daylight anywhere else on the slope other than the originally assumed slope toe.

## Introduction Cont'

- On another hand, the failure plane conditions such as roughness, smoothness as well as infill material.
- Which have a direct effect on the overall friction can result in the variation of the base friction and the inter-block friction
- This eventually leads to the notion of the possible variation of the base and inter-block friction resistances ( $\phi_p \neq \phi_d$ )
- In the case of toppling failure, specifically slide-head toppling



## Design Charts Concept

- Determination of slide-head toppling is two-fold
- Thus toppling and sliding of blocks

**With the two limit equilibrium equations as follows;**

$$P_{n-1,t} = \frac{W_n (y_n \sin \psi_c - \Delta x \cos \psi_c) + P_n (M_n - \Delta x \tan \phi_d)}{L_n} \dots (1)$$

$$P_{n-1,s} - P_n = - \frac{W_n (\cos \psi_c \tan \phi_p - \sin \psi_c)}{(1 - \tan \phi_p \tan \phi_d)} \dots (2)$$

## Design Charts Concept Cont'

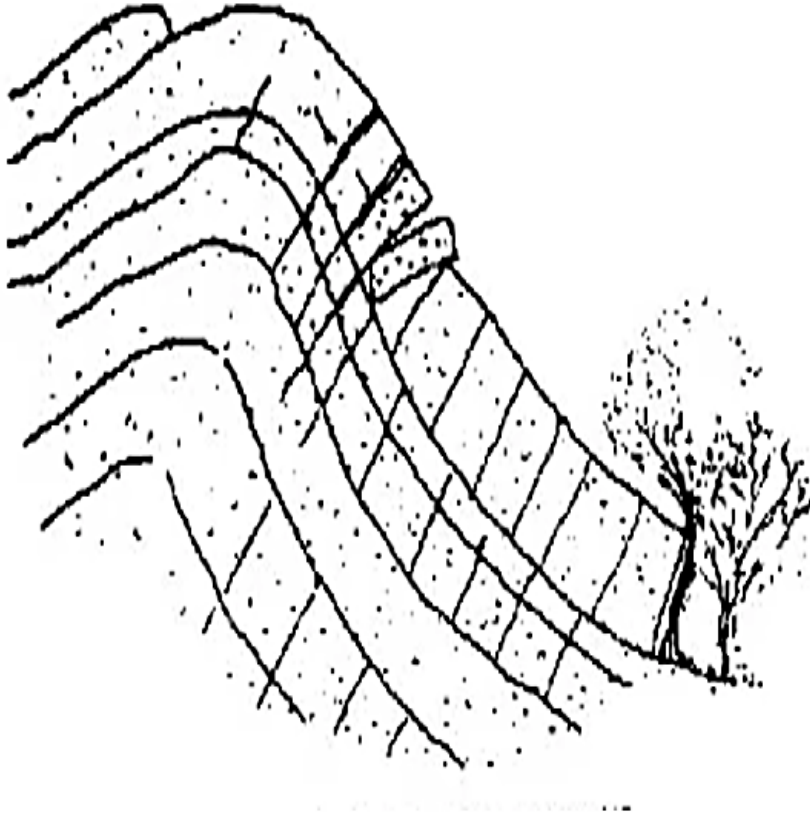
- Where;  $P_{n-1,t}$ ,  $P_{n-1,s}$  and  $P_n$ , represent forces
- $\psi_c$  denotes initial failure plane (before counter-tilting)
- $\phi_p$  denotes the base friction resistance
- $\phi_d$  denotes the inter-block friction resistance
- $W_n$  denotes the weight of the rock block under consideration

Figure below illustrates the general slide-head toppling failure concept as well as the application of the two equations highlighted above.



# Design Charts Concept Cont'

a



b

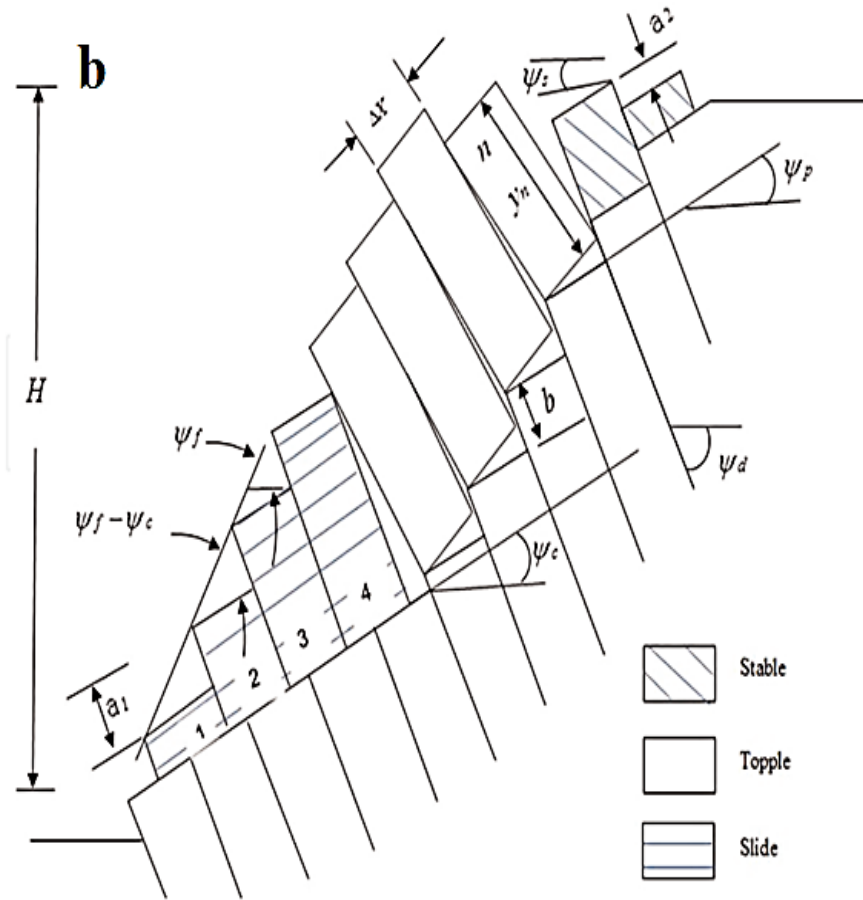


Fig 1: Conceptualisation of Slide-head toppling failure: (a) The general slide-head toppling failure concept as originally illustrated by Goodman and Bray (1976); (b) Schematic illustration of the toppling and sliding sections of a slope undergoing slide-head toppling (Bowa and Samson, 2022; Bowa and Xia, 2018).



## Design Charts Concept Cont'

- The governing equation for the previously developed charts as described by Zambak (1983) were based entirely on Equation 1.
- Additionally, as seen from Equation 1, only the inter-block friction resistance affects the equation.
- As such, It is therefore, not possible to develop charts based on the proposed governing principle ( $\phi_p \neq \phi_d$ ) for Equation 1.

## Therefore,

- Taking into consideration of Equation 2 and bearing in mind of the possibility for the initial failure plane to undergo counter-tilting within the rockmass, the following modifications to the equation can be made;



## Design Charts Concept Cont'

$$P_n - P_{n-1,s} = \frac{W_n(\cos\psi_c \tan\phi_c - \sin\psi_c)}{(1 - \tan\phi_c \tan\phi_d)}$$

$$P_{n-1,s} = P_n - W_n(\zeta) \dots (3)$$

Where;

$$\zeta = \frac{(\cos\psi_c \tan\phi_c - \sin\psi_c)}{(1 - \tan\phi_c \tan\phi_d)} \dots (4)$$

From Equation 4, it is therefore noted that zeta ( $\zeta$ ) varies with respect to three variables namely; the base frictional resistance ( $\phi_c$ ), the inter-block frictional resistance ( $\phi_d$ ) and the weak plane angle ( $\psi_c$ ) within the rockmass



## Development of 2D Design Charts

- These developed slide-head toppling charts can be applied in both situations where there is existence of the counter-tilting of the failure plane or not
- Equations 3 and 4 govern the development of these design charts
- In the case of counter-tilting of the weak plane, in Equation 4, as the weak plane counter-tilts from the initial failure plane ( $\psi_c$ ), can be designated as  $\psi_p$ .
- Hence, rewriting Equation 4 with respect to counter-tilting of the weak plane yields Equation 5 as below;

$$\zeta = \frac{(\cos\psi_p \tan\phi_c - \sin\psi_p)}{(1 - \tan\phi_c \tan\phi_d)} \dots (5)$$



## Development of 2D Design Charts.. Cont'

- From Equation 5 it can be perceived that the limiting value for the counter-tilted angle for all positive values of  $\zeta$  is the base friction resistance.
- Thus;

$$\zeta = \begin{cases} \lim_{\psi_c \rightarrow \phi_c} 0 \\ \lim_{\psi_p \rightarrow \phi_c} 0 \end{cases} \dots (6)$$

- The charts have been developed using MATLAB software (The MathWorks Inc. 2016)
- The parameter zeta ( $\zeta$ ) is plotted against inter-block friction resistance ( $\phi_d$ ) for various values of counter-tilted weak plane angles ( $\psi_p$ ) with reference to a constant base frictional resistance ( $\phi_c$ ) for each chart.



## Development of 2D Design Charts.. Cont'

- The basal friction resistance therefore determines to what extent the counter-tilted angle can extend to from the original failure plane for all positive values of Zeta.
- For weak planes dipping at angles above the limiting basal friction ( $\phi_c$ ), it is observed that the design chart can be applied as a 'mirror of itself'.
- From Equation 6, it is noted that the zeta ( $\zeta$ ) values become zero when the counter-tilted angle is or approaches the value of basal friction.
- Therefore, by calculation, it has been observed that each amount of addition to the failure plane angle equal to the basal friction is a negative replica of the same amount deducted from the basal friction angle.
- It is basically a negative mirror of its own/itself.



## Development of 2D Design Charts.. Cont'

- For instance, the zeta ( $\zeta$ ) values of weak plane of  $30^\circ$  are an exact mirror of the weak plane of  $40^\circ$  in the negative form for the  $35^\circ$  basal friction chart.
- In summary, the following Equations 7 to 9 are true and govern the preceding observations;

$$\zeta, (\phi_c - x) = -\zeta, (\phi_c + x) \dots (7)$$

For any integer values of  $x$ ,

Where  $(\phi_c - x)$  and  $(\phi_c + x)$  are designated values of  $\psi_p$

Furthermore,

$$\zeta \geq 0, \text{ for } \psi_p \leq \phi_c \dots (8)$$

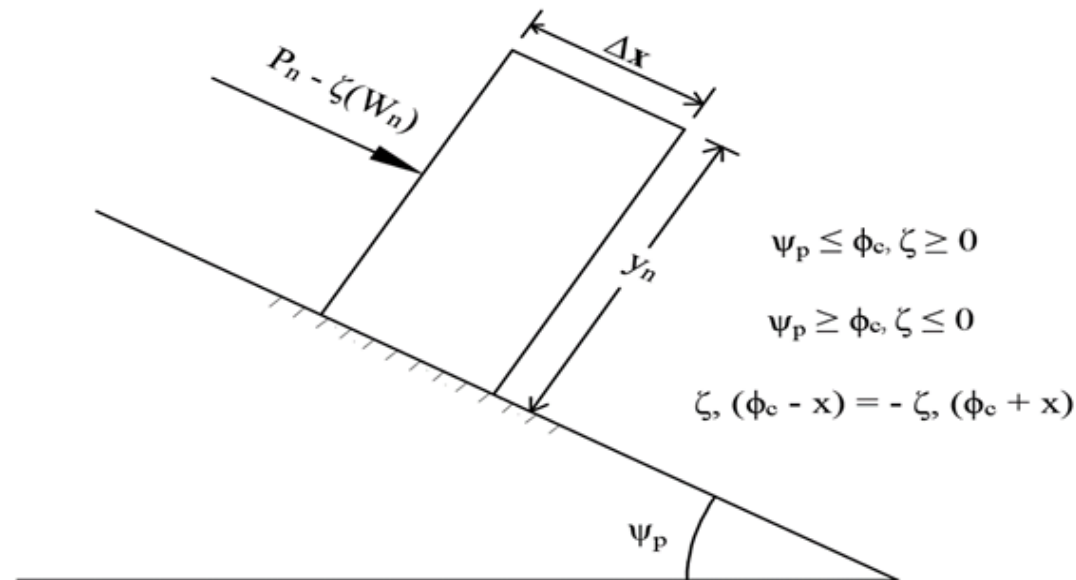


## Development of 2D Design Charts.. Cont'

And

$$\zeta \leq 0, \text{ for } \psi_p \geq \phi_c \dots (9)$$

Figure 2 depicts the principle on which the developed charts have been based on with reference to the sliding rigid rock blocks and the equations that govern it described above;



# Developed Charts

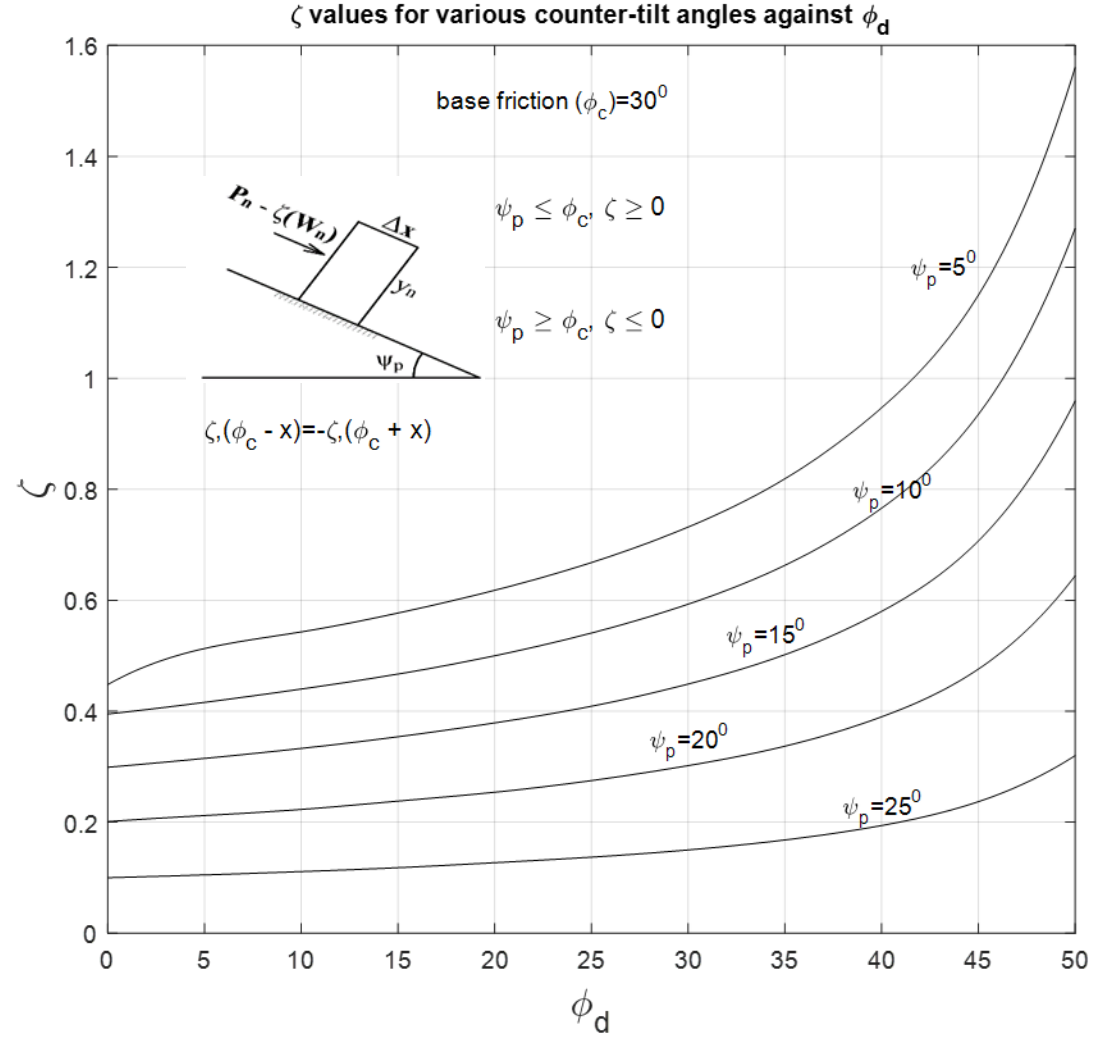
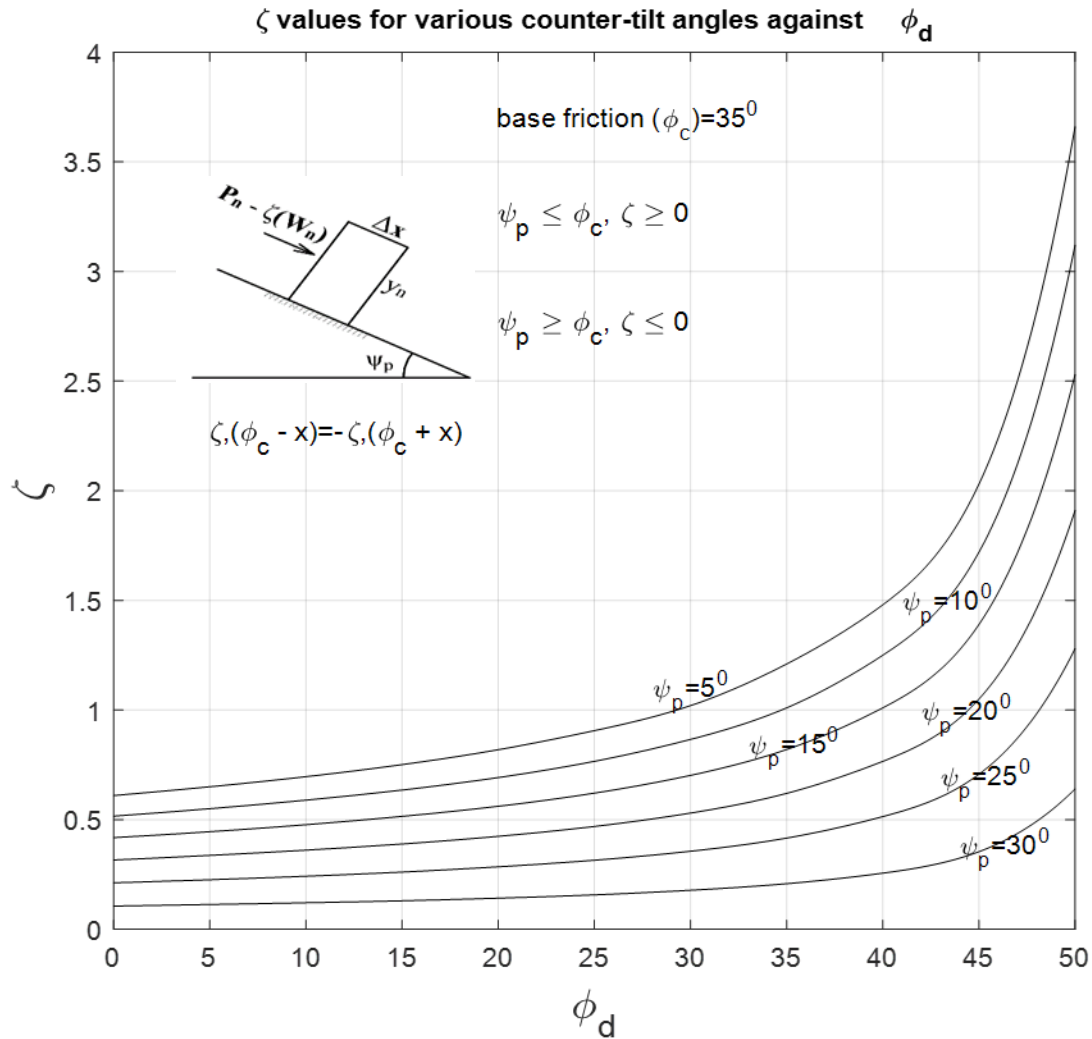
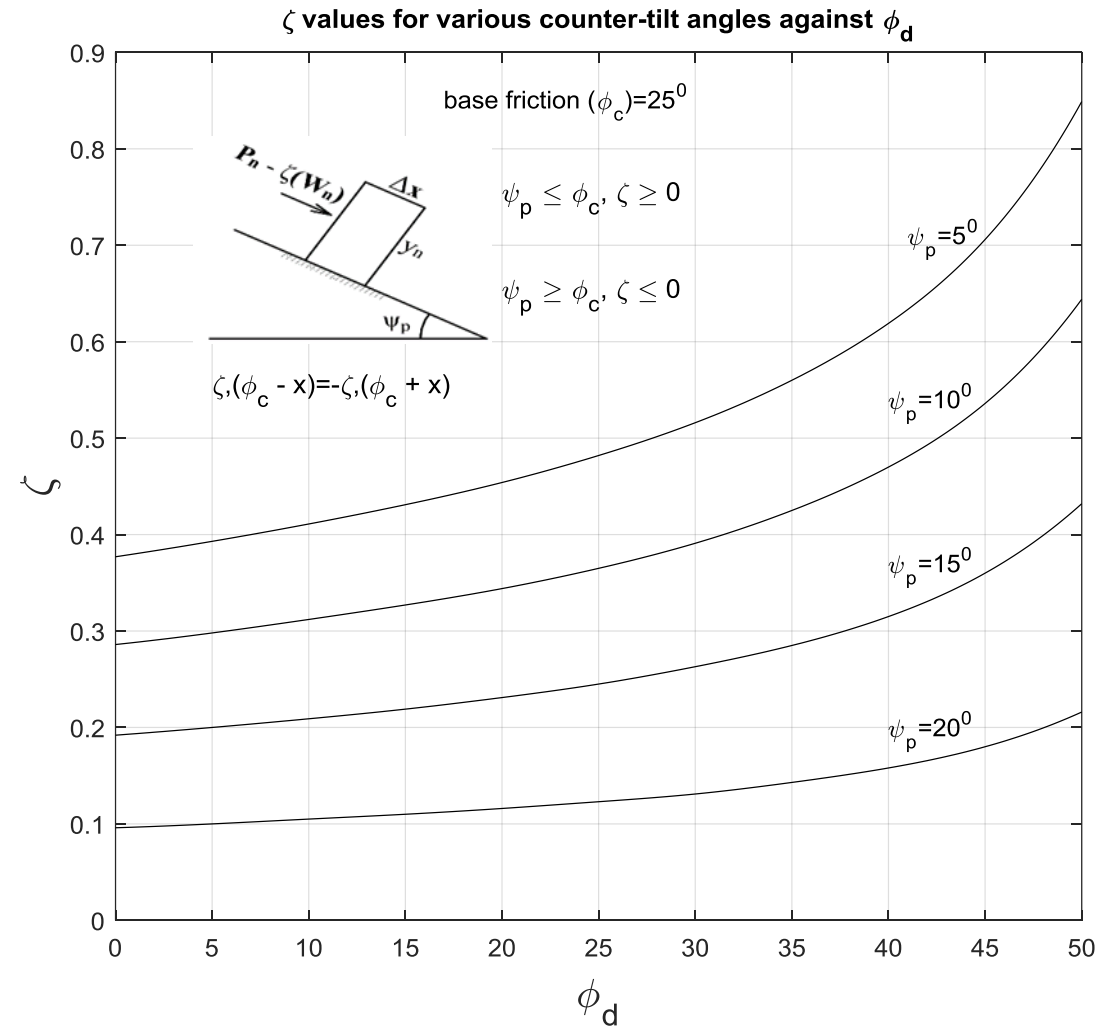


Fig. 3: Developed 2-dimensional design chart for various weak plane and counter-tilt angles ( $\psi_p$ ) against inter-block friction resistance ( $\phi_d$ ); (i) basal friction angle of  $35^\circ$ , (ii) basal friction angle of  $30^\circ$ , (iii) basal friction angle of  $25^\circ$



# Developed Charts



## Discussion and Conclusion

- The development of the charts has been done on the premise that the base friction is different from the inter-block friction resistance ( $\phi_p \neq \phi_d$ )
- The charts aim to provide a preliminary check on the possibility for sliding of rock blocks under slide-head toppling in counter-tilted rock slopes. But can also be applied in situations of normal failure planes.
- The charts can be utilised in two ways:
  - I. In situations that support Equation 6 as provided in the preceding sections
  - II. Where the base friction is less than the failure plane angle for slide-head toppling (equation 7)
- To obtain Zeta ( $\zeta$ ), the two friction resistances (base and inter-block friction) need to be provided and properly defined.



## Discussion and Conclusion

- For curiosity's sake, the principle on which these charts have been developed ( $\phi_p \neq \phi_d$ ) using Equation (2) may not be possible to apply for the development of similar charts based on Equation (1).
- The variables and /or parameters that govern a rock block to undergo toppling on a slope do not depend on/include the base friction.

## Way forward

- Further research is planned to simulate conditions of counter-tilting of the failure plane within the rockmass as highlighted in the preceding paragraphs.
- This will be achieved through the use of both numerical and experimental modelling.



*The End*

THANK YOU FOR YOUR ATTENTION.

